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# On the permissibility of approximating irregular cavity geometries by rectangular boxes and cylinders

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## ABSTRACT

A general procedure for the three dimensional analysis of a gas cavity by the concept of four pole parameters has been developed by Kim and Soedel [1]; the procedure was later extended by Lai and Soedel [2] to two dimensional analysis, which is more efficient in computation than the 3-D analysis method and can solve some geometries which are difficult to solve by the 3-D method. In this study, the above procedures are applied to a realistic compressor head with irregular corners, flow obstacles, and a bullet shaped muffler by idealized rectangular box and circular cylinder approximations. The experimental tests are performed, discussed in this paper, and compared to the simulation results. The comparisons demonstrate the limitations of an overly idealized theoretical shape analysis for practical applications. Certain response frequencies which shift between idealized theory and experimental data are traced to certain acoustic path constrictions in the compressor head and the muffler which result in a geometric deviation from the idealized box and bullet shapes. It is concluded that while idealized shape approximations are useful for a qualitative understanding of gas pulsation muffling behavior, precise geometric modeling is needed for precise predictions. Or one has to rely on transfer function measurements.

## ANALYTICAL MODELS

Four pole parameters are very useful for the analysis of acoustic systems. Basic discussions of the concept and derivation of four poles of various acoustic elements are found in references [3, 4, 5, 6]. A gas cavity can be represented by the following format:

$$\begin{bmatrix} Q_1 \\ P_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} Q_2 \\ P_2 \end{bmatrix}, \quad (1)$$

where  $Q_1, P_1$  are the input volume velocity and acoustic pressure,  $Q_2, P_2$  are the output volume velocity and acoustic pressure.  $A, B, C, D$  are the so-called four pole parameters.

### Compressor Head

For the compressor head as shown in Figure 1, the 2-D model [2, 7] can be applied since the thickness is very small compared to the shortest wavelength of interest. Therefore, the four poles can be obtained:

$$A = \frac{f_2(\tilde{r}_2, \omega)}{f_1(\tilde{r}_2, \omega)}, \quad (2)$$

$$B = \frac{1}{f_1(\tilde{r}_2, \omega)}, \quad (3)$$

$$C = -f_2(\tilde{r}_1, \omega) + \frac{f_1(\tilde{r}_1, \omega)}{f_1(\tilde{r}_2, \omega)} f_2(\tilde{r}_2, \omega), \quad (4)$$

$$D = \frac{f_1(\tilde{r}_1, \omega)}{f_1(\tilde{r}_2, \omega)}, \quad (5)$$

where  $\tilde{r}_1, \tilde{r}_2$  are the input and output positions respectively.  $f_1$  and  $f_2$  are pressure response functions, which are

$$f_1(\tilde{r}, \omega) = \sum_{k=0}^{\infty} \frac{j\omega \rho c^2 P_k(\tilde{r}_1) P_k(\tilde{r})}{N_k [(\omega_k^2 - \omega^2) + 2j\omega \omega_k \xi_k]}, \quad (6)$$

$$f_2(\tilde{r}, \omega) = \sum_{k=0}^{\infty} \frac{j\omega \rho c^2 P_k(\tilde{r}_2) P_k(\tilde{r})}{N_k [(\omega_k^2 - \omega^2) + 2j\omega \omega_k \xi_k]}, \quad (7)$$

where Cartesian coordinates have to be used and

$$N_k = \int_x \int_y P_k^2(x, y) h dx dy. \quad (8)$$

### Compressor Muffler

A bullet shape compressor muffler as shown in Figure 2 is also analyzed by the four pole parameters. This system can be divided into two subsystems: a straight pipe and a cylinder with round ends.

The four pole parameters of the straight pipe are obtained from the 1-D model [4]:

$$A_1 = \cosh(\gamma L), \quad (9)$$

$$B_1 = \frac{j\omega A_o}{\rho c^2 \gamma} \sinh(\gamma L), \quad (10)$$

$$C_1 = \frac{\rho c^2 \gamma}{j\omega A_o} \sinh(\gamma L), \quad (11)$$

$$D_1 = \cosh(\gamma L). \quad (12)$$

$A_o$  is the cross section area of the pipe,  $\gamma = \frac{\xi \sqrt{\omega}}{2cd} + j\frac{\omega}{c}$ ,  $\xi$  the damping ratio,  $d$  the diameter of the pipe, and  $L$  the length of this pipe.

The bullet shape cylinder is approximated in this case by a circular cylinder. Because the cross sectional dimension is not small, the 3-D model [1] has to be utilized instead. The four pole parameters are the same as equations 2 to 5 except that

$$N_k = \int_r \int_\theta \int_z P_k^2(r, \theta, z) r dr d\theta dz. \quad (13)$$

The overall equivalent four poles of the total gas cavity are then obtained by utilizing the cascading property of four pole matrices:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \quad (14)$$

### Transfer Function

In practical applications, the exhaust pipe leading all the way to the condenser is usually assumed as an anechoic termination [8, 9]. Therefore, the transfer function of a gas cavity is obtained:

$$\frac{P_2}{Q_1} = \frac{1}{A \frac{S}{\rho c} + B}, \quad (15)$$

where  $S$  is the cross section area of the exhaust pipe.

## TRANSFER FUNCTION MEASUREMENT

The measurement of the transfer functions or transmission loss (usually used in auto industry) can be found in different references [9, 10, 11, 12, 13]. The measurements for the compressor head and muffler are shown in Figures 1 and 2 respectively. A piston at the cavity entrance is attached to a dynamic shaker to

generate sound. The acceleration of this oscillating piston can be measured and then the input volume velocity is obtained by integration. Note that the piston should be long enough to keep a good alignment while oscillating. A microphone is installed at the exit of the cavity to measure the output acoustic pressure. Therefore, the transfer function between the output acoustic pressure and input volume velocity can be measured. Since a very long exhaust pipe is used to simulate the anechoic termination [14], the two-microphone technique is not necessary in this case.

## RESULTS AND DISCUSSION

The results from simulations and experiments for the compressor head and bullet muffler were compared in Figures 3 and 4, respectively. Both the sinusoidal sweep excitation and random noise excitation were performed [9]. The analytical models basically matched the characteristics of gas pulsation in the head cavity and muffler. However, some response frequencies shifted between idealized models and experimental data. The primary reason is the geometric deviation of the actual compressor head, which possesses many flow restrictions like valves and bolt stands, from an idealized box shape. Similarly, the bullet shape of the muffler is not a circular cylinder in reality.

In order to understand the deviations, the transfer functions of three simple geometries were simulated and compared.

### A Pipe with a Narrow Restriction

The transfer function of a pipe ( input on the left, output on the right ) with a narrow restriction as shown in Figure 5 was simulated and compared to the case of a straight pipe of the same length. Since the purpose of this investigation was only to understand the deviation instead of accurate simulation, the 1-D model as mentioned in previous section was employed. All the cross sectional modes were neglected. The four pole parameters can be obtained and the transfer function was solved along with the anechoic exit assumption. The results showed that the response frequencies shift to either lower or higher frequency ranges depending on the magnitude of the flow restriction. But there is no specific rule as to which way the frequencies will shift.

### A Pipe with Two Narrow Restrictions

Another case, which is more similar to the compressor head with two valve settings inside, is a pipe containing two narrow restrictions as shown in Figure 6. For different sizes of flow restrictions, the resonances in the transfer functions deviated from that of a straight pipe of the same length considerably, showing that these kind of restrictions cannot be ignored.

### A Pipe with Two Small Ends

The last case of study is a pipe with two narrow ends, which are similar to the bullet shape muffler in geometry. The comparison of the transfer functions for this pipe and a straight pipe of the same length is illustrated in Figure 7. The response frequencies shift either way due to different small ends.

## CONCLUSION

The concept of four pole parameters was implemented for the analysis of a compressor head and bullet shape muffler. It was found that the geometric deviation due to the small flow restrictions in cavities can change the response frequencies of gas pulsations. Therefore, the idealized shape approximations are only suitable for a fast and qualitative understanding. To obtain more accurate prediction, the small flow obstacles have to be taken into account, especially for the cavities whose sizes are not large compared to the small obstacles.

## ACKNOWLEDGMENTS

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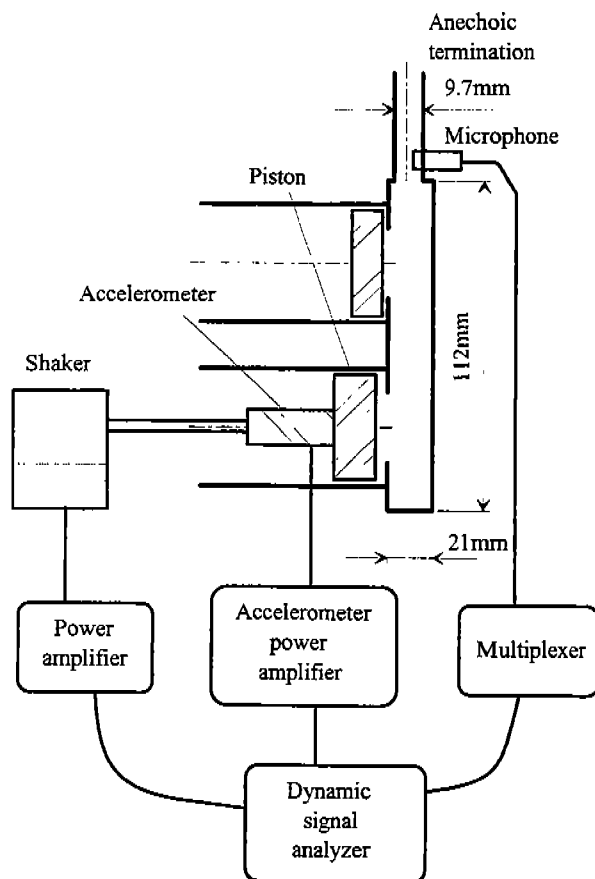


Figure 1: Experimental setup for compressor head transfer function measurement.

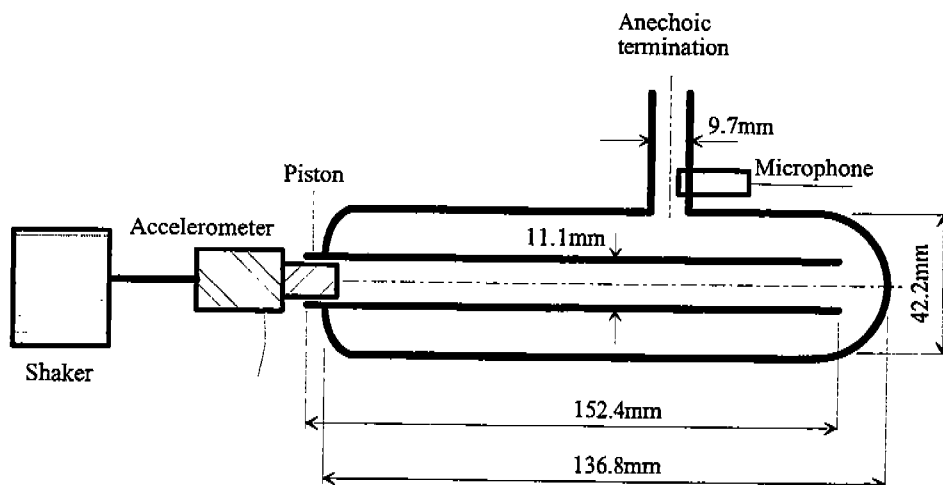


Figure 2: Experimental setup for compressor muffler transfer function measurement.

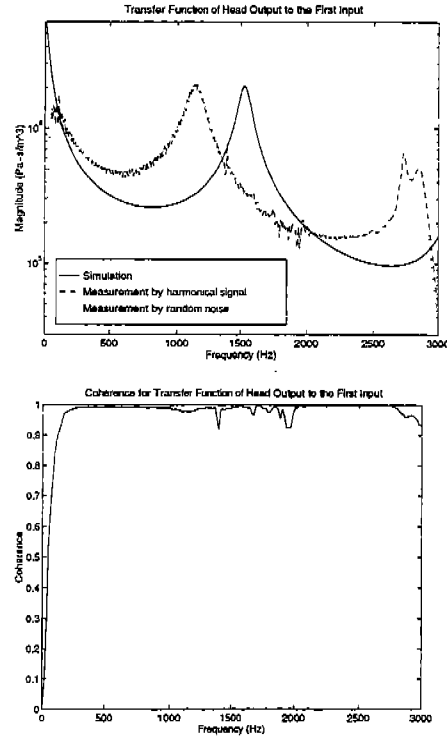


Figure 3: Compressor head cavity transfer function. —, theory; — — —, experiment ( harmonic ); ···, experiment ( random ).

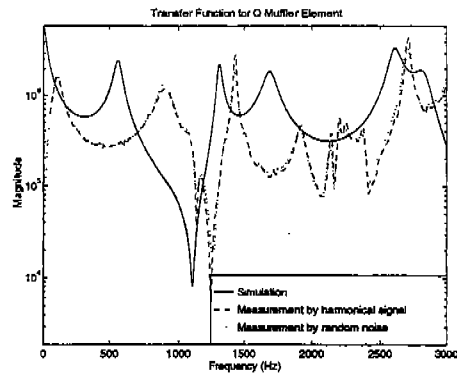


Figure 4: Compressor muffler transfer function. —, theory; — — —, experiment ( harmonic ); ···, experiment ( random ).

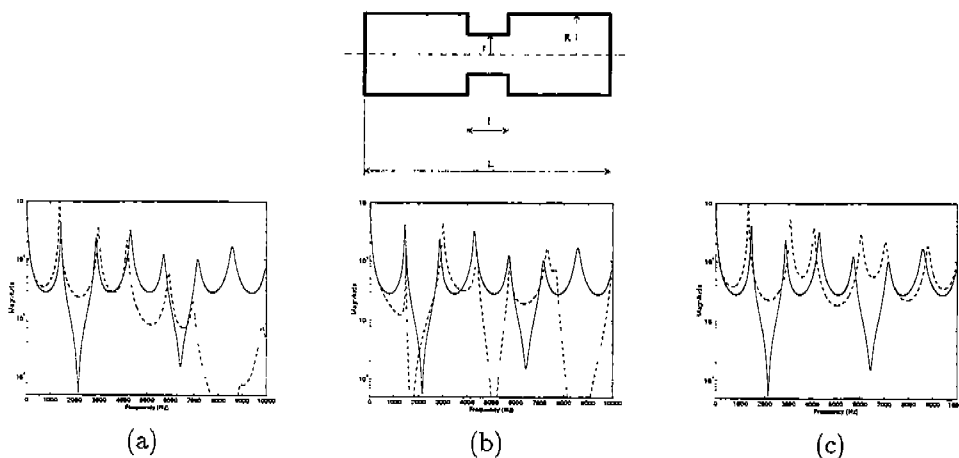


Figure 5: Parametric transfer function comparison. —, straight pipe; ---, pipe with a small restriction.  $R = 0.02m$ ,  $r = 0.01m$ ;  $L = 0.12m$ , (a)  $l = 0.010m$ , (b)  $l = 0.015m$ , (c)  $l = 0.020m$ .

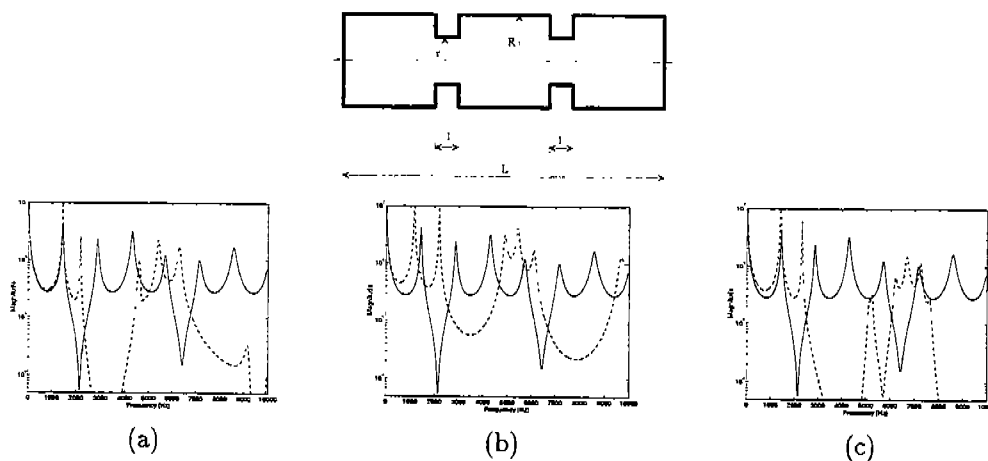


Figure 6: Parametric transfer function comparison. —, straight pipe; ---, pipe with two small restrictions.  $R = 0.02m$ ,  $r = 0.01m$ ;  $L = 0.12m$ , (a)  $l = 0.005m$ , (b)  $l = 0.010m$ , (c)  $l = 0.015m$ .

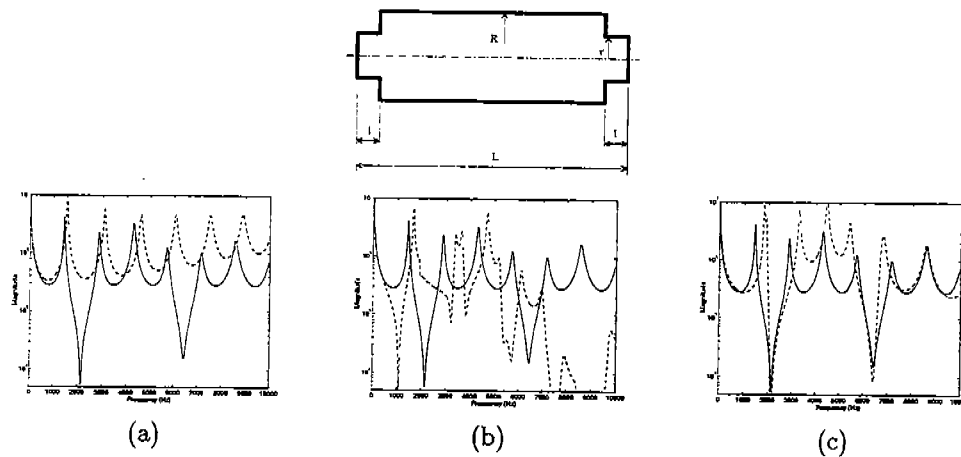


Figure 7: Parametric transfer function comparison. —, straight pipe; ---, pipe with two narrow ends.  $R = 0.02m$ ,  $r = 0.01m$ ;  $L = 0.12m$ , (a)  $l = 0.010m$ , (b)  $l = 0.015m$ , (c)  $l = 0.020m$ .